Reply to 'Intuitionism Without Intuition: Against the Phenomenological Account'

Mark van Atten

Abstract

I am grateful to Dr Ronzitti for having taken the time to consider a phenomenological account of intuitionism, and of choice sequences in particular, and write down her critical reflections on it. But I'm afraid that 'Intuitionism Without Intuition: Against the Phenomenological Account' does not do much to advance the discussion. My principal reasons for thinking so are threefold:

   1. The paper's presentation of the phenomenological account of choice sequences is incomplete in an essential way.

   2. Of the two imagined (and negative) reactions to the phenomenological account, the first one is incoherent and the second highly implausible.

   3. The paper's view on the relations between mathematics and philosophy leaves a mathematics that is a mere formalism and that is unrelated to the content of intuitionistic mathematics as developed by Brouwer and his followers. The label 'intuitionism without intuition' that Dr Ronzitti gives to her proposed mathematics is misleading at best.

1. The paper's presentation of the phenomenological account of choice sequences

In her description and analysis of the phenomenological account, Dr Ronzitti refers to an article ('Brouwer, as Never Read by Husserl', van Atten 2003) and a book of mine (*Brouwer Meets Husserl*, van Atten 2007). Both the article (which is short, being the text of a talk at a conference) and the book answer the question if choice sequences are mathematical objects in two stages. First, are choice sequences objects at all? Second, are they specifically mathematical objects? I answer `yes' to the first question because a phenomenological analysis supplies me with an individuation criterion, and `yes' to the second question because, I argue, the objects thus construed are such that what is proved about them once is proved forever, they are purely formal in the sense that they do not depend on sense data, and they do not

lead to truths that cannot be shared intersubjectively (in van Atten 2007, these are sections 6.3.1, 6.3.2, and 6.3.3, respectively).

In 'Intuitionism Without Intuition', however, the second stage is left out completely. Having presented just the first stage, Dr Ronzitti then concludes that thereby we have my answer to the question why choice sequences are mathematical objects. This is clearest when, on page 87, she quotes from my article (van Atten 2003: 12):

> I suggest that what remains invariant is the character of the sequence as a developing sequence, a development that started at a particular point in time.

and then comments that 'This is the phenomenological solution to the problem of the legitimation of ips as mathematical objects'.

But it is not; and Dr Ronzitti never enters into the reasons I give for holding that these objects are not only objects but, specifically, mathematical objects. She stops quoting from the article 'Brouwer, as never read by Husserl' right before that paper's explicit transition to a discussion of the question why choice sequences are specifically mathematical objects; a discussion that begins on its page 13, final paragraph, and continues for more than two pages. And the corresponding, more detailed discussion of that question in section 6.3 of *Brouwer Meets Husserl* is never cited or referred to either.

More generally, it is astonishing that in a paper that aims to be a critical discussion of the phenomenological account, of the longest and most detailed of the two sources it uses, the book *Brouwer Meets Husserl*, exactly one page is ever referred to. This is its page 22 (which Dr Ronzitti refers to on page 85) of the chapter 'The Original Positions'; thus, the further chapters 'The Phenomenological Incorrectness of the Original Arguments', 'The Constitution of Choice Sequences', and 'Application: An Argument for Weak Continuity' are passed over.

2. The two imagined reactions to the phenomenological account

In section 3 of her paper, Dr Ronzitti evaluates the impact of the phenomeno-logical account by asking to whom it may be directed and what they would think of it. To my mind, she is wrong to hold (page 89) that the phenomeno-logical account of exactly why choice sequences are mathematical objects cannot be meant for those who already accept both phenomenology and intuitionism. After all, one's acceptance of phenomenology or intuitionism or both may be based on general considerations, or on positive experience with

them related to other objects than choice sequences. That leaves wide open the possibility that a particular phenomenological analysis of certain objects in intuitionistic mathematics may deepen one's understanding of them and lead to the discernment of further principles valid for them. (An example that, although referred to in a quotation on page 84, Dr Ronzitti otherwise ignores, even though it seems to be the kind of application she is asking for, is the phenomenological grounding of the Weak Continuity Principle in chapter 7 of *Brouwer Meets Husserl*.)

The first case that Dr Ronzitti considers, the case of 'those who do not accept the notion of a sequence that develops over time as mathematically meaningful while perhaps remaining neutral as to which philosophical approach (if any) to embrace' (page 89), is incoherent. For not accepting the notion of such a sequence as mathematically meaningful is just as much taking a philosophical stance as accepting that they are. Naturally, individual mathematicians may decide not to enter into that philosophical discussion in their own intellectual activity; but that does not amount to making the philosophical question go away. It is just a division of labour.

The imagined reaction in the second case that Dr Ronzitti considers, the case of 'those who are not sympathetic with the phenomenological approach while being willing to look into intuitionistic mathematics' (page 90), is highly implausible. The key claim is this: 'But, if before actually doing intuitionistic mathematics they come to believe that, as the phenomenologist claims, a phenomenological introduction is necessary, they will simply stop in their attempt.' But why on earth would someone who has little sympathy for phenomenology suddenly believe the phenomenologist on this point? And anyone who thinks that there is nothing to phenomenology is free to look for other reasons, be they philosophical or not, to adopt choice sequences. There is no reason at all, in this case, to let oneself be stopped by phenomenologists, whatever they shout.

3. The paper's view on the
relations between mathematics and philosophy

Dr Ronzitti holds that mathematics can be done without philosophical worries over objects, and without philosophical worries more generally, because we would 'just learn how to make mathematical reasonings' (page 83); one just needs 'to be acquainted with the adapted principles of reasoning' (page 89); and mathematicians 'simply look for such principles' (page 90).

Of course an individual may choose to adopt that attitude; but in the abstract that resolves nothing. What Dr Ronzitti does not seem to realise is

that the view she mobilises here leaves a mathematics that, in its lack of concern with objects (or perhaps the term 'neutral' should be used here, as on page 88) and hence with questions of meaning and truth, amounts to a mere formalism. For one thing, this view leaves it wholly unclear what the `new, useful, mathematical insight' she is striving for (page 83) would be insight into. For another, that mere formalism has nothing to do with intuitionistic mathematics as it was conceived by Brouwer and developed by Heyting, Troelstra, Van Dalen, Veldman, and others. Their intuitionistic mathematics is evidently based on philosophical considerations, namely considerations on the nature of mental acts, of mental objects, of evidence, of truth, of freedom, of intuition. Surely, then, the label 'intuitionism without intuition' that Dr Ronzitti gives to her proposed mathematics is misleading at best. As (in effect) a mere formalism, it has nothing to do with other varieties of constructivism either, nor, for that matter, with classical mathematics.

By cutting the ties between philosophical understanding and mathematical principles, the approach advocated by Dr Ronzitti leaves it inexplicable exactly why the choices of objects and principles governing them in intuitionistic mathematics so far have been fruitful for their purpose; and it blocks a road to motivating the introduction of further objects and new principles. And in cutting these ties, the paper sets up a false dichotomy between 'the mathematician' and 'the phenomenologist' (or 'the philosopher'). Thus, it is overlooked throughout the paper that there have been a number of individuals who engaged both in mathematical thinking and in philosophical thinking, both of the highest order, in such a way that the two informed each other. One finds examples among constructive as well as classical mathematicians: Cantor, Hilbert, Brouwer, Weyl, Gödel, Martin-Löf. To take Gödel's case: he was a classical set theorist and a phenomenologist at the same time, who argued that a phenomenological foundation of classical mathematics should be looked for. Indeed, when on page 83 Dr Ronzitti says that the claim is that 'unlike in the case of classical mathematics, a philosophical justification is needed' to engage in intuitionistic mathematics, she is mistaken: phenomenologists claim that classical mathematics stands in need of such a justification just as much.

Let me conclude with a reference to well known literature. To my mind, the view on the relations between mathematics and philosophy expounded in 'Intuitionism Without Intuition' has been discredited effectively by Georg Kreisel in his paper 'Informal rigour and completeness proofs' (Kreisel 1967); the interested reader is referred to pages 140 to 143 (first two lines).

References

Kreisel, G. (1967).　Informal rigour and completeness proofs. In Imre Lakatos (ed.). *Problems in the Philosophy of Mathematics*. North-Holland, pp. 138–157.

van Atten, M. (2003). Brouwer, as Never Read by Husserl. *Synthese* 137, Issue 1, pp. 3–19.

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