Alf Ross and Jørgen Jørgensen on Reasoning about Directives and Norms

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Abstract
This paper is a study of the foundations of deontic logic in the light of Alf Ross's paradox of disjunctive directives and Jørgen Jørgensen's problem about logical relations among imperatives ("Jørgensen's dilemma"). It analyzes performative and assertoric utterances of deontic sentences and the distinction between norms (directives) and normative (deontic) propositions. The relation of logical consequence among normative propositions can be defined in the usual way in terms of the concept of truth, and it is argued that the logic of normative propositions (as defined here) can serve as the logic of norms.

Keywords: Directive, Jørgensen's Dilemma, Norm, Normative Proposition, Ross's Paradox.

1. Ross's Paradox

Students of deontic logic know Alf Ross mainly from a problem called "Ross's paradox". This problem has generated an extensive literature and engendered both foundational discussions and formal developments in deontic logic. It arises in the following way.

Since the late Middle Ages philosophers have treated deontic logic (the logic of normative discourse) as a branch of modal logic (Knuuttila 1981; 1993, ch. 5). For example, Leibniz called the basic deontic categories (the obligatory, the permitted, and the prohibited) "legal modalities" (*iuris modalia*), and observed that the basic principles of modal logic hold for the legal modalities. Moreover, he suggested that deontic modalities can be defined in terms of the alethic modalities ("the modalities of being"): the permitted (*licitum*) can be defined as "what is possible for a good man to do", and the obligatory (*debitum*) as "what is necessary for a good man to do". (Leibniz 1671/1930, pp. 465–66; Poser 1969, pp. 16–19.) Consequently the basic principles of modal logic may be expected to hold for deontic as well as alethic modalities.

A simple system of deontic modal logic is obtained by adding to propositional logic two modal axioms

(OD) $\mathbf{O}A \rightarrow \neg\mathbf{O}\neg A$
and
(OK) $\mathbf{O}(A \rightarrow B) \rightarrow (\mathbf{O}A \rightarrow \mathbf{O}B)$,

where **O** represents deontic or legal necessity, and the rule of "deontic necessitation", according to which **O**A is provable if A is provable:

(RON) If $\vdash A$, then $\vdash \mathbf{O}A$.

The concepts of deontic possibility (**P**) and impossibility (**F**) are defined by the formulas

(P) $\mathbf{P}A \leftrightarrow \neg\mathbf{O}\neg A$
and
(F) $\mathbf{F}A \leftrightarrow \mathbf{O}\neg A$.

Principle (OD) may be termed the consistency principle for O-sentences. According to (P), schema (OD) can be written in the form

(1.1) $\mathbf{O}A \rightarrow \mathbf{P}A$.

A deontic sentence of the form '**O**A' can be interpreted in different ways, for example:

(i) A is obligatory (its is obligatory that A),
(ii) A is a duty,
(iii) It ought to be the case that A,
or as
(iv) A is required (it is required that A).

These interpretations of **O**A[1] are not synonymous, but in deontic logic they are usually assumed to obey similar logical principles. The concept of requirement can be regarded as the most general interpretation of **O**; (i)–(iii) express different

[1] If it is clear from the context that an expression, formula, or sentence is mentioned rather then used, I often write the expression without quotation marks, for example, **O**A instead of '**O**A'.

forms of requirement. **P**A can be taken to mean that A is permitted or that it may be the case that A, and **F**A means that A is prohibited or illegal. The readings 'A is obligatory' and 'A is permitted' for **O**A and **P**A suggest that A is a general act-description or act-predicate, and in his first (1951) system of deontic logic G. H. von Wright understood the schematic letters in this way; thus his 1951 system was based on the logic of act-predicates rather than propositional logic. However, most authors, including Alf Ross, have regarded deontic operators as propositional operators, and I am assuming here that the expressions A, B, etc., represent propositions which can be true or false.[2] Thus the expressions 'A is obligatory' and 'A is permitted' should be regarded as abbreviations of 'it is obligatory that A' and 'it is permitted that A'.

The modal system defined by these axioms and rules is called the system KD or briefly D,[3] and is often called "the standard system of deontic logic", abbreviated 'SDL' (Hilpinen 2001a, pp. 160–62). According to this account of deontic modalities, the main difference between deontic and alethic modalities is that the principle

(T) $\mathbf{N}A \rightarrow A$,

where **N** is a symbol for the concept of necessity, does not hold for deontic necessity (the concept of obligatoriness).

The valid inference rules of the standard system include the consequence principles (monotonicity conditions; cf. Chellas (1980), pp. 190–91, 234)

(ROM) If $\vdash A \rightarrow B$, then $\vdash \mathbf{O}A \rightarrow \mathbf{O}B$,

(ROK) If $\vdash A_1 \,\&...\&\, A_n \rightarrow B$, then $\vdash \mathbf{O}A_1 \&...\& \mathbf{O}A_n \rightarrow \mathbf{O}B$ $(n = 0,1,2,...)$[4],

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[2] For some difficulties concerning von Wright's (1951) interpretation of the schematic letters in deontic formulas, see Føllesdal and Hilpinen (1971), pp. 13–14. Cf. Stenius (1963), p. 248, and von Wright (1968), p. 16.

[3] For the standard method of naming modal systems, see Chellas (1980), pp. 131–33.

[4] (ROK) with n = 1 is the rule (ROM), and n = 0 gives the rule of "deontic necessitation" (RO). Deontic systems which contain (DO) and (ROK) are called *normal* systems. (ROK) and (DO) form a sufficient axiomatic basis for the standard deontic logic; see Chellas (1980), pp. 114–15, 190–91. Thus the standard system of deontic logic may also be called the "normal system of deontic logic".

and the principle intensionality,

(ROE) If $\vdash A \leftrightarrow B$, then $\vdash \mathbf{O}A \leftrightarrow \mathbf{O}B$,

which permits the substitution of provably equivalent formulas for each other in deontic contexts. According to (ROM),

(1.2) $\mathbf{O}A \rightarrow \mathbf{O}(A \vee B)$

is a valid principle of deontic logic. For example, the following conditional is an instance of (1.2):

(1.3) If Peter ought to mail a letter, then Peter ought mail or burn it.

According to Ross (1941, p. 62), (1.3) does not represent a logically valid inference; therefore (1.2) is not a valid conditional. Thus the consequence principles expressed by (ROM) and (ROK) do not hold for deontic modalities. Many philosophers have been inclined to agree with Ross's judgment; for example, according to G. H. von Wright, it "seems odd" to reason that if one ought mail a letter, one ought to mail or burn it (1968, pp. 20–21). He has suggested that the invalidity of schema (1.2) might be one of the disanalogies between deontic and alethic modalities (von Wright 1983, pp. 103–104). Formula (1.2) is "paradoxical" in the sense that its validity has been thought to be an intuitively unacceptable consequence of the view that the standard system of deontic logic, or any modal system in which rules (ROM) and (ROK) hold, is a good partial systematization of the logic of normative discourse. On the other hand, it has been argued that if the O-operator is interpreted in the proper way, the Ross formula (1.2) is perfectly acceptable, and does not constitute a problem for the conception of deontic logic as a branch of modal logic. (See Hansson 1969/1971, pp.130–132; Føllesdal and Hilpinen 1971, pp. 21–22.) The system of deontic logic outlined above is based in the view that the possibilities open to an agent (courses of events, scenarios, situations, or possible worlds) in a given situation or possible world-state u are divided into permitted or legal (normatively acceptable) possibilities and illegal (normatively unacceptable) possibilities. The former possibilities may be said to constitute the *field of permissibility* of the agent or norm-subject in situation u.[5] An O-sentence **O**A is true in a given situation u if and only if A is true in all possibilities which

[5] David Lewis (1979, p. 163) uses the expression "sphere of permissibility".

are acceptable relative to u, **F**A is true in u if and only if A is not true in any legal possibility, and **P**A is true in u if and only if A is holds in some legal possibility. If A is an action description, **P**A means that there is some acceptable way of performing A, and **F**A means that all possible ways of performing A are illegal. It is clear that this interpretation of the O-sentences makes schema (1.2) valid: if Peter mails the letter in all legal possibilities, then he obviously either mails or burns the letter in all such possibilities.

Many philosophers have sought to dispel the appearance of a paradox in Ross's example in a way which is compatible with the validity of (1.3) and the schema (1.2). (See Hare 1967, pp. 313–14, Føllesdal and Hilpinen 1971, pp. 21–22.) A and B are possible ways of fulfilling the disjunctive obligation A $\vee$ B, and some ways of fulfilling the obligation may be unacceptable. Bengt Hansson has observed that the good or acceptable ways of fulfilling an obligation depend on the system of obligations as a whole: "even if the good ways of fulfilling an obligation are not determined by that very obligation, they are determined by the set of all obligations." (Hansson 1969/1971, p. 132). In Ross's example, mailing a letter is an acceptable way of fulfilling the obligation to mail a letter or burn it, but burning it is an unacceptable way, because it is excluded by the obligation to mail the letter. As Hare (1967, p. 313) has observed, "we cannot in general be sure of fulfilling commands [obligations] by fulfilling other commands which are inferable from them." In other words, **O**A is true in a situation u only if A is true in all legal possibilities open to the agent in u, but not all possibilities in which A is true are necessarily legal.[6]

2. Jørgensen's Dilemma

Alf Ross did not formulate the Ross paradox in the way described above. When he wrote his paper (1941), the old view of deontic logic as a theory of normative modalities had been forgotten and was not recovered until the 1950's by the work of G. H. von Wright (1951), Oskar Becker (1952, pp. 40–50), Stig Kanger (1957/71), Jaakko Hintikka (1957), and others. Ross's observation that (1.2) should not be regarded as a valid principle of the logic of norms was part of the discussion of the possibility of a logic of imperatives, or more generally, the

[6] For some recent attempts to explain why reasoning in accordance with the Ross formula (2.1) appears paradoxical, see Hintikka (1979), Hilpinen (1984), and Danielsson (2005, 2007). Ross's paradox is one of several alleged deontic paradoxes related to the consequence principle (ROM); these puzzles include the Paradox of the Good Samaritan and the Paradox of Epistemic Obligation; see Åqvist (1967). For a survey of various paradoxes in deontic logic, see Hilpinen and McNamara (2013), pp. 58–97.

possibility of a logic of normative discourse. In the late 1930's and early 1940's Ross and a number of other philosophers were interested in a puzzle concerning the logic of imperatives and directives ("Forderungssätze"; see Dubislav 1937), formulated by Ross's countryman Jørgen Jørgensen as follows (1938, p. 284): According to the standard conception of logical inference (entailment), "a conclusion follows logically from certain premisses if and only if it cannot be the case that the premisses are true and the conclusion is false."

> Consequently it is essential for logical inference that its premisses and its conclusion are sentences which can be true or false. Imperative sentences do not fulfill this condition. Therefore imperative sentences cannot function as the premisses or conclusions of logical inferences. And therefore it is in principle impossible to justify an imperative by means of logical reasoning. (Jørgensen 1938, p. 184; my translation.)

On the other hand, Jørgensen notes:

> It seems equally evident ... that there are inferences in which one or both premisses as well as the conclusion are imperative sentences, and yet the conclusion seems just as inescapable as the conclusion in any syllogism containing sentences in the indicative mood only. (Jørgensen 1937–38, p. 290.)

Here is one of Jørgensen's examples (Ibid., p. 290):

> Love your neighbor as yourself!
> Love yourself!
> (Therefore:) Love your neighbor!

This seems to be an instance of valid reasoning with imperatives.

Ross calls this problem "Jørgensen's dilemma" (1941, p. 55; 1968, p. 139). This problem has continued to engage philosophers until the present, for example, G. H. von Wright published in the 1990's a paper entitled 'Is There a Logic of Norms?' (1996), and David Makinson (1999) has called Jørgensen's dilemma "a fundamental problem of deontic logic".[7] Ross's original formulation of the problem of disjunctive obligation was part of his discussion

[7] For a survey of the recent discussion on Jørgensen's dilemma, see Hanson (2013).

of this problem, the question about the very possibility of a logic of imperatives and other normative utterances.

Jørgensen (1937–38, p. 288) says that he takes the word 'imperative' to mean imperative sentences, that is, "sentences in which the main verb is in the imperative mood." Ross (1941, p. 55; 1958, pp. 8–9) points out that in the study of the logic of imperatives, the linguistic or grammatical form of sentences is unimportant: the logic of imperatives (or directives) is concerned with sentences used in a certain way, that is, for expressing "an immediate demand for action". Such demands or requests can be expressed by means of "linguistically indicative" sentences as well as sentences in the imperative mood; for example, the directives of a legal code are often expressed in the indicative form[8]. For the purposes of the present discussion such sentences count as "imperatives". Following Ross, I shall use the expression 'imperative' below to refer to a certain speech act type (a type of illocutionary act), not to the grammatical mood of a sentence. Thus the expression 'imperative' is regarded here as interchangeable with 'directive' or 'command'. (It is clear that Jørgensen's dilemma concerns all kinds of normative utterances, not only "immediate demands for action.") In the same way, Jørgensen's "indicatives" (indicative or descriptive sentences) should be taken to mean *propositions* or *statements*, that is, sentences which can be used for stating facts or describing the world. Moreover, the words 'imperative', 'directive', and 'command' exhibit the product-process ambiguity: they can mean the content of the speech act (what is commanded) or the act itself, for example, the act of commanding the addressee to do something.

Jørgensen suggests two possible ways of solving the problem about the possibility of the logic of norms (1937–38, p. 290).

(1) First, we may widen our concept of valid inference so that it need not be defined in terms of the concept of truth. According to this proposal, logic can be said to have "a wider reach than truth" (von Wright 1957, p. vii).

[8] According to Ross (1968, p. 37), legal language provides examples of directives "which are expressed neither in the imperative mood nor in sentences containing deontic expressions. The Danish criminal code says that whoever kills another man is imprisoned for five years to life; and the constitution states that the King orders the promulgation and execution of statutes." See also Ross (1958, pp. 8–9). Here is an example from my personal experience: When I was a schoolboy and at one time misbehaved in the class, my teacher gave me the following order: "Hilpinen will leave the class and will stay outside the whole hour!"

(2) Secondly, we can try to solve the puzzle by defining the validity of imperative reasoning indirectly, in terms of the truth-values of indicatives (statements) which are related to the imperatives in a suitable way. In this way of dealing with the puzzle, the (apparent) logical relations among imperatives are regarded as being constituted by relations among certain statements associated with the imperatives.

In the first way of tackling Jørgensen's dilemma, the logical relations among directives are defined in terms of some semantic feature which can be regarded as analogous to the concept of truth. (Cf. Grue-Sörensen 1939, p. 197.) Albert Hofstadter and J. C. C. McKinsey (1939, p. 447; cf. Ross 1941, p. 60) have suggested that the concept of *satisfaction* can play such a semantic role: an imperative or a directive cannot be said to be true or false, but it can be satisfied or not satisfied by the actions of the addressee. An imperative is satisfied if (and only if) what is commanded is the case. For example, the directive 'Peter, mail the letter!' is satisfied if and only if Peter mails the letter. In another variant of this approach, logical relationships among directives are defined in terms of the "validity" of a directive or a norm so that the concept of validity plays the same role in the analysis of directive reasoning as the concept truth in "indicative" reasoning. This use of the word 'valid' must be distinguished from the concept validity applied to the evaluation of an argument; this distinction can be made explicit by calling the former notion 'norm validity' and the latter 'logical validity'. According to Ross, our intuitive conception of valid normative reasoning is based on the validity of norms and directives. (Ross 1941, p. 59; Ross 1968, p. 172.)[9] The validity of a norm means its " 'existence' or 'being in force' – however these expressions are to be understood", as Ross puts it (1968, p 175; see also Kelsen 1960/1967, pp. 10–15).[10]

In his own attempt to solve the puzzle, Jørgensen chooses the second approach, following a proposal made by Walter Dubislav. According to Dubislav (1937, p. 341), every directive ("Forderungssatz") D is related to a certain statement ("Behauptungssatz") s(D) in such a way that our judgments

[9] "The logical deduction of [a directive] I_2 from I_1 then means that I_2 has objective validity in case I_1 has objective validity." (Ross 1941, p. 59.)

[10] Alchourrón and Martino (1990, p. 48) have argued that the meaning of deontic concepts can be expressed directly by the rules for their use in deductive reasoning; thus logical relations among norms (directives, imperatives) can be understood by means of such rules, and this does not necessarily require a characterization in terms of the concept of truth or norm validity or some other semantic concept.

about the logical relationships among directives are determined by the logical relationships among the corresponding statements: A directive F can be inferred from D if and only if the statement s(F) associated with F is a logical consequence of s(D). What we take to be logical relationships among imperatives are really relationships among the statements associated with the imperatives. According to Jørgensen, any imperative (or directive) can be analyzed into two factors which he calls the imperative factor and the indicative factor. The former element indicates that something is commanded or requested, and the latter element describes what is commanded. (Jørgensen 1937–38, p. 291.) Ross calls Jørgensen's "indicative factor" (what is demanded) the *theme* of a directive: "The theme of [a] demand consists of a certain fact, or a state, or an activity." (Ross 1941, p. 56) In his book (1968, p. 34) he calls the content of a directive its *topic*. The topic of the directive

(2.1) Peter, mail this letter!

can be expressed by the proposition ("indicative")

(2.2) Peter mails this letter

(Ross 1941, p. 56) or, to indicate that the proposition is not being asserted, by the action description

(2.3) Peter to mail this letter.

If the imperative factor (or directive factor) is expressed by the exclamation mark '**!**', (2.1) has (according to Jørgensen and Ross) the form

(2.4) **!**(Peter to mail this letter).[11]

It should be noted that if the topic (i.e., the "indicative factor") of a directive is expressed by an infinitive clause with a subject, as in (2.4), it should be regarded

[11] The distinction between the content (or the topic) and the directive factor of a directive is a special case of the distinction between the illocutionary character and the content of a speech act. R. M. Hare has expressed the same distinction by the words 'phrastic' (for the content) and 'neustic' (for the illocutionary character); See Hare (1952, p. 18), and Ross (1968, p. 18).

as a proposition.[12] If D = **!**A, where A is a proposition, Jørgensen takes A to be the "indicative" (proposition or statement) s(D) which determines the logical relations of D to other directives, and he regards imperative reasoning (or reasoning about directives) as reasoning about their propositional contents:

(2.5) An imperative **!**B is said to be derivable from **!**A if and only if the statement B is derivable from A.

In this way "the imperative factor is so to speak put outside the brackets much as the assertion-sign in the ordinary logic [logic of statements], and the logical operations are only performed within the brackets." (Jørgensen 1937–38 , p. 292.) Thus the logic of imperatives is reduced to the logic of statements for which the concept of logical consequence can be defined in the usual way. Ross (1941, p. 57) observed that according to the Jørgensen-Dubislav proposal, "there seems to be no reason for, and hardly any possibility of, constructing a special 'logic of imperatives'." [13]

Ross points out that these attempts to escape Jørgensen's dilemma lead to the same result: According to both proposals, "the logical element refers solely to the fulfilment of the demand, or rather to the indicative sentences expressing the theme of the demand as real, or the demand as fulfilled." (Ross 1941, p. 61.) (2.2) is the topic of (2.1), and (2.2) entails

(2.6) Peter mails or burns this letter;

thus the Dubislav-Jørgensen solution validates the problematic (or objectionable) principle (1.2). Moreover, it is clear that if (2.4) is satisfied, the disjunctive directive

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[12] This is a convenient way to represent unasserted propositions, cf. Peirce (1976, p. 248). Following Hare (1952, p. 17), Ross uses the formulation "Shutting of the door by Peter", see (1968, p. 34). Stig Kanger (1972, p. 122) makes a distinction between an act and acting: "An act is an action expressed by a noun phrase, for instance, moon-walking, murder, handshaking, etc. Acting is action expressed by a sentence, for example, p walks on the moon, p kills someone, p and q shake hands, ..., etc." The action descriptions 'Peter to mail this letter' and 'Shutting of the door by Peter' express actions in the latter, propositional sense.

[13] The "expressive conception of norms" formulated and defended by Carlos Alchourrón and Eugenio Bulygin (1981) resembles Dubislav's and Jørgensen's semantics of directives. According to the expressive conception, the difference between statements (assertions) and commands arises only on the pragmatic level of the use of language; there is no such difference on the semantic level. (Alchourrón and Bulygin 1981, pp. 96–97; 1993, p. 273.)

(2.7) **!**(Peter to mail or burn this letter)

is also satisfied; thus the analysis of the imperative reasoning in terms of the concept of norm-satisfaction also validates (1.2). Such an analysis is equivalent to the Dubislav-Jørgensen proposal. This is one of the reasons why Ross rejects both attempts to solve Jørgensen's dilemma; both lead to the same objectionable result:

> From the imperative I(x) we may infer the imperative I($x \vee y$), e.g. from: slip the letter into the letter-box! we may infer, slip the letter into the letter-box or burn it! It will be seen that, interpreted as a satisfaction-function, this inference is unimpeachable: If the first imperative is satisfied (if the letter has been slipped into the letter-box), then the other imperative has been satisfied (it is then true that the letter has been slipped into the letter-box, or it has been burnt). But it is equally obvious that this inference is not immediately conceived to be logically valid. (Ross 1941, p. 62; 'I' is used here as a sign of an imperative, and x and y are propositional letters.)

According to Ross,

> It is surely not a logic of such content [the logic of satisfaction] which we have in mind in case of the practical inferences which seem immediately evident to us. The immediate feeling of evidence does not refer to the satisfaction of the imperative, but rather to something like the "validity" or the "existence" of the imperative, no matter how those expressions are understood. (1941, p. 61.)

An attempt to analyze the logic of norms as a logic of norm-satisfaction leads to difficulties which are independent of the question of the validity of the schema (2.1), for example, as Stig Kanger (1957/1971, p. 49) has shown, in such a logic it is difficult to represent the logical relationship between obligations and permissions in an intelligible way. (Cf. Hilpinen 2001b, p.134.)

In the method of analyzing imperative inference considered above, the indicative (statement) s(D) associated with a given directive is regarded as a proposition which expresses the content of the directive. Jørgensen observes that the logic of imperatives could also be reduced to the logic of statements by taking s(D) to be a statement which says that "the ordered actions are to be

performed, resp. the wished state of affairs is to be produced." According to this method, the command 'Close the door!' is correlated with the indicative sentence 'The door is to be closed.' (1937–38, p. 292). Thus the indicative counterpart of the command (2.1) may be expressed by the *deontic proposition*

(2.8) Peter is required to mail this letter,

which may also be written in the form 'It is required that Peter mail this letter'. If the requirement (or obligation) expressed or created by a command is expressed by the deontic O-operator, (2.8) has the form

(2.9a) **O**(Peter mails this letter)
or
(2.9b) **O**(Peter to mail this letter).

According to this construal of the logic of directives,

(2.10) s(**!**A) = **O**A.

Here **O**A is a normative proposition which is true or false, and **!**A is a directive which is regarded as not having a truth-value. In this method Jørgensen's dilemma is solved by representing the logic of directives as the logic of deontic (normative) propositions.[14] I shall call this method Jørgensen's second method of associating indicatives with imperatives. Normative propositions obey the rules of "ordinary logic" in the sense that the concept of logical consequence can be defined in the usual way by means of the concept of truth (cf. Jørgensen 1937–38, p. 292), but it is important to note that the logical constants of this logic include operators for the basic normative concepts (the obligatory, the permitted, and the prohibited), and the "ordinary logic" of normative propositions is therefore a branch of intensional (modal) logic.[15]

[14] I am using the expressions 'normative proposition' and 'deontic proposition' synonymously.

[15] For some recent approaches to the logic of imperatives (commands), see Vranas (2011) and Hansen (2013, 2014).

3. Directives, Norms, and Normative Propositions

Deontic sentences can be used in two ways: for expressing genuine norms or directives and for making normative statements. The latter are descriptive propositions which state that something is obligatory, permitted or prohibited according to a given system of norms (Stenius 1963, p. 250, Føllesdal and Hilpinen 1971, p. 8, Hansson 1969/1971, p. 123, Alchourrón and Bulygin 1971, p. 121). For example, the deontic sentence 'Motor vehicles ought to use the right-hand side of a road' can be regarded as a directive addressed to drivers, or as a proposition which gives information about the traffic code of some country. If it is regarded as a statement about the U.S. traffic regulations, it is a true statement, but understood as a statement about the U.K. regulations, it is false. Normative propositions, unlike the norms themselves, are true or false, and the logical relationships among normative propositions can therefore be understood in the usual way in terms of the concept of truth. The distinction between the normative and the descriptive or *assertoric* use of deontic sentences goes back (at least) to Jeremy Bentham, who distinguished between *authoritative* and *unauthoritative* books of "expository jurisprudence". A book is authoritative when it is composed by the legislator himself; unauthoritative, when it is the work of any other person (1789/1943, pp. 323–24). Ingemar Hedenius (1941, pp. 65–66) makes a similar distinction between "genuine" ("äkta") and "spurious" ("oäkta") legal sentences, and Hans Kelsen distinguishes an "authentic" interpretation of law by legal organs from jurisprudential ("nonauthentic") interpretation: only the former can create law (1960/1967, p. 355).[16]

The possibility of using norm sentences assertorically, to state that according to a norm system which is in force in a given situation, things ought to be in a certain way or something ought to be done, and normatively, to create norms, does not mean that strictly speaking, there is only a logic of normative statements, but no logic of directives or norms, as some philosophers have concluded (Hedenius 1941, pp. 120–130; Moritz 1954, pp. 82–83; Williams 1963). The distinction between two ways of using norm sentences can be understood as a distinction between two kinds of *utterances*

[16] Kelsen (1960/1967, p. 72) formulates the same distinction as a distinction between a legal norm (*Rechtsnorm*) and a legal statement (*Rechtssatz*); legal statements describe law without possessing the legal authority of genuine norms. However, it should be noted that the expression 'Rechtssatz' (or 'Rechtsnormsatz') is also used to refer to "authoritative" legal propositions; see Weinberger (1988), p. 86. According to Weinberger, a *Rechtssatz* should be understood as a linguistic formulation of a *Rechtsnorm*.

of normative propositions. A normative proposition, for example, the proposition that vehicles must use the right side of the road (which is true or false), can be *uttered* assertorically, to give information about an independently existing system of traffic regulations, or *performatively* or normatively, to give a command and thus create a norm (bring about an obligation). (See Kamp 1979. pp. 263–64; Raz 1980, pp. 45, 47.) [17] In the latter case, the utterance of the proposition in the appropriate circumstances (by a proper norm authority) has normative force, and is sufficient to make the proposition true; in the former case the truth of the proposition depends on whether it fits a norm system whose content is independent of the utterance in question.[18] The utterer of the proposition can make the intended normative force of the utterance evident by expressing the proposition in the (grammatically) imperative mood or by adding to the utterance the word 'hereby', as in 'You are hereby required to mail this letter'. In the case of legal norms and directives, normative utterances include the written inscriptions (occurrences) of norm sentences in authoritative legal texts and documents. Thus the prescriptive-descriptive distinction is a distinction between two kinds of utterances, performative (or normative) and assertoric utterances, and both kinds of utterances can have the same content, i.e., express the same normative proposition which can be true or false.

Hans Kamp has observed that the assertoric use of deontic sentences depends on their performative use (1979, p. 264). Performative utterances of normative propositions constitute their own "truth-makers", and they also constitute the truth-makers of assertoric (descriptive) utterances of the same

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[17] Ross (1972) rejects the use of the expression 'performative' in this context, and prefers to speak about the normative use of norm sentences; see Ross (1972), pp. 211–12.

[18] See David Lewis (1979, p. 166). Lewis calls the norm authority the Master and the addressee the Slave, and characterizes the semantics of commands and permissions in the following way:

> The [normative] sentence may be used to command: the Master says it to the Slave, his purpose is to control the Slave's actions by changing the sphere of permissibility, and truthfulness is automatic because the sphere adjusts itself so that saying makes it so. The sentence may be used to inform: either the speaker is not the Master or the hearer is not the Slave, the speaker's purpose is to impart information to one who does not yet possess it, and truthfulness is not automatic.

The sphere of permissibility consists of the legal or normatively acceptable possibilities open to the addressee (i.e., the Slave).

propositions.[19] Moreover, the content of a descriptive utterance of a normative proposition (e.g., that Peter is required to tell the truth) must be the same as the content of the normative utterance 'Peter, tell the truth!'; otherwise an assertoric utterance would not give a correct representation of the norm in question. In their performative use, the function of O- and F-sentences (obligation and prohibition-sentences) is to restrict the range of normatively acceptable options ("the field of permissibility") available to a norm-subject (the addressee), whereas permission sentences have the opposite effect: they enlarge the set of normatively acceptable possibilitites. (An O-sentence **O**A excludes all possibilities in which A does not hold.) Hans Kamp has put forward the following principle concerning the performative and assertoric uses of permission sentences (1979, p. 264):

(3.1) An assertoric utterance of a permission sentence **P**A in a context u is true if and only if the field of permissibility open to the agent in u already contains all the possibilities that a performative use of **P**A would have added to the field if it had not already contained them.

In the same way, the use of O-sentences can be regarded as being governed by the following rule:

(3.2) An assertoric utterance of an O-sentence **O**A in a context u is true if and only if the field of permissibility open to the agent in u already excludes all the possibilities that a performative use of **O**A would have excluded if they had not already been excluded.

Kamp has also observed that it is not always clear whether a deontic sentence is used performatively or assertorically. However, if the assertoric use of deontic sentences is governed by (3.1) and (3.2), assertoric utterances of deontic sentences can guide and direct the agent's actions in the same way as their performative utterances. For example, in the case of a permission sentence, "either the utterance is a performative and creates a number of new options, or else it is an assertion; but then if it really is appropriate it must be true; and its truth then guarantees that these very same options already exist" (Kamp 1979, p. 264). The practical consequences of the utterance for the addressee are the same in both cases.

[19] For the concept of a truth-maker, see Mulligan et al. (1984) and Armstrong (2004).

If normative and assertoric utterances of a given deontic sentence have the same content, a normative proposition, there is no difference between the logic of norms (directives) and the logic of normative statements. According to this view, Jørgensen (1937/38) and Ross (1941) made a mistake when they took the content of a normative utterance '**!**(Peter to mail this letter)' to be the proposition that Peter mail the letter: the content of the utterance is the *deontic* proposition '**O**(Peter to mail this letter)'. Therefore the formula (2.10) above,

s(**!**A) = **O**A,

shows the *content* of the directive **!**A, not only a statement or "indicative" which "corresponds" to the directive **!**A. (Cf. Ross 1968, p. 182. Note that according to the view proposed here, the sign '**!**' should be regarded as a sign of a kind of utterance rather than as a sign of a sentence type.) Ross himself seems to accept this conception of the content of directive utterances when he observes that as a directive, **O**A means "the directive which directs that the subject is under an obligation (or is commanded, is advised) to behave in such a way that A becomes true." (1968, p. 155). As was observed above, a speaker can indicate that he is using the sentence 'You are required to mail this letter' performatively rather than assertorically by adding to it the word 'hereby':

(3.3) You are hereby required to mail this letter.

Adding the word 'hereby' to the utterance does not change its logical properties. Such a normative utterance constitutes its own truth-maker.

Ross defines the distinction between directives (i.e., prescriptive norm sentences) and normative propositions (statements) as follows (1968, p. 182):

> O(*p*) stands for a directive which prescribes an 'obligation' to act in such a way that the proposition *p* becomes true. It is usual to say that to a directive there corresponds an indicative. This may, however, mean two different things. The corresponding indicative might be *p*; or it might be O(*p*) ε N (which expresses the fact that O(*p*) exists or is *in force* in the normative order N, or, if O(*p*) is a personal directive, the fact that O(*p*) has been *advanced* in an interpersonal situation).

Ross is using 'O' here as a sign of a directive; *p* is a propositional letter. (For a more extensive discussion of the distinction, see Ross 1958, pp. 8–11.) He

uses 'F[O(*p*)]' as an abbreviation of 'O*p* ε N'; thus 'F[O(*p*)]' is the "indicative" counterpart of the directive O(*p*). Taking *p* as the relevant indicative would lead to the "logic of satisfaction" rejected by Ross. According to Ross's interpretation (or translation), 'F[O(*p*)]' is a statement about the directive O(*p*) and a normative system N: it states that O(*p*) belongs to N. This way of transforming directives to statements does not help to solve Jørgensen's dilemma, because the membership of N presumably depends on the deductive relationships among directives. This is not a problem for Ross, because he defines valid normative reasoning in terms of the validity conditions of norms and directives, and assumes that the validity conditions of norms determine the truth-conditions of normative statements (cf. Ross 1968, p. 183).[20] According to Jørgensen's second method of associating statements with directives, the statement to be associated with the directive O(*p*) is the deontic proposition **O***p* (i.e., it is required that *p*), or, if the reference to a normative system N is made explicit,

According to N, **O***p*.

The reference to the system N can be regarded as part of the situation or context u in which deontic propositions are evaluated; thus we arrive at the simple formulation of the semantics of deontic propositions given at the end of Section 1.

A normative system is obviously not a system of utterances, but a system of norms whose content can be expressed by deontic propositions. The system derives its normative force from the performative (prescriptive) utterances of norm sentences which identify the system and tie it to reality.[21]

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[20] G. H. von Wright (1963, p. 134) seems to hold a similar view when he writes:

> The laws (principles, rules), which are peculiar to [the logic of descriptively interpreted expressions], concern the logical properties of the norms themselves, which are then reflected in logical properties of norm-propositions. Thus, in a sense, the 'basis' of Deontic Logic is a logical theory of prescriptively interpreted O- and P-expressions.

According to the view of directives and deontic propositions adopted in the present paper, von Wright's view should be expressed by saying that the normative utterances of deontic propositions are the truth-makers of their assertoric utterances.

[21] According to this way of identifying a normative system, identity of content is not a sufficient condition of the identity of normative systems: even if N_1 and N_2 contain the same normative propositions, they are distinct systems if they are based on different normative utterances; cf Raz (1980), p. 128 n. 1.

Such utterances are the ultimate truth-makers of deontic propositions. If a normative system is regarded as a deductively closed system, it contains, in addition to the normative propositions expressed or formulated in the authoritative utterances, the logical consequences of such propositions.

As the content of a performative utterance, a normative proposition does not refer to an independently existing normative system; in this respect the present conception of deontic (normative) propositions differs from the characterization given by Stenius (1963), Ross (1968), and Alchourrón and Bulygin (1993). The sense of a normative proposition **O**A can be grasped independently of the system to which it belongs; thus the same normative proposition can belong to different systems.

A norm authority can obviously utter conflicting normative propositions, for example, **O**A and **O**¬A, or even **O**A and **P**¬A (i.e., ¬**O**A). If a norm system is determined and identified by the authoritative utterances of normative propositions, this suggests that a normative system may contain conflicting norms, and that the consistency principle (OD) does not hold for normative propositions. If the authority utters **O**A and then **P**¬A (or both simultaneously), the latter directive cancels or derogates the former, and the two utterances together fail to create any norm. Some authors have argued that this need not happen in the first case, in other words, that the two utterances (**O**A and **O**¬A, or **O**A and **O**B, where A and B are incompatible) create a normative system which contains conflicting norms. From this it has been concluded that the principle (OD) does not hold for deontic propositions. (See Stenius (1963), p. 254; Schotch and Jennings (1981), pp. 154–156.)[22]

Ross rejects this argument. It is no doubt possible that a normative authority can utter incompatible directives; in this sense conflicting norms can co-exist in a norm system. In his early paper (1941) Ross considered a psychological interpretation of the concept of norm-validity, according to which a directive O*p* is valid when either the norm-giver or the norm-subject (the addressee) is in a certain psychological state, namely, when the former wants the directive to be followed, or when the latter is willing to comply with the directive. If the

[22] Brian Chellas defines "minimal deontic logic" as a system which consists of the rule (ROM) and the principle

¬**O**(A & ¬A).

(Note that Chellas calls this principle 'OD' and the consistency principle of SDL 'OD*'; see Chellas 1980, 201–202.). The Ross formula (2.1) is valid in Chellas's minimal deontic logic.

validity of a directive is understood in this way, then the principles of the logic of directives turn out to be empirical laws, and the possibility of conflicts between directives is, as Ross observed, a psychological and empirical question (1941, pp. 58–60, 62–63). Ross's psychological interpretation of the logic of norms was subsequently criticized by Ota Weinberger (1957, pp. 109–111, 124–126), and in his book (1968) Ross agrees with Weinberger's criticism and rejects such an interpretation. He now regards the principles of the logic of norms to be normative and regulative principles which lay down the necessary conditions of rational normative discourse, or the conditions which must be satisfied if utterances are to fulfill their function of directing people's actions. Ross characterizes the principles of deontic logic as follows (1968, p. 178):

> The principles of deontic logic, in my opinion, are postulates defining directive speech, that is, conditions to be satisfied if the speech is to be able to carry out the function of directing human behavior. If these postulates are violated it becomes impossible to distinguish between what is posed in directive discourse, that is, conceived as what 'ought' to be real ..., and what is not posed; and it is consequently equally impossible to distinguish between what can be accepted as valid and what cannot.

Ross (1968, p. 183) observes:

> Stating F(O(*p*)) (e.g., that *A* has ordered *B* to pick up his hat) is not stating only the linguistic fact that *A* at some time in relation to *B* uttered some words; not only the psychological fact that *A* has done so with certain intentions. It is also an interpretation of these facts as having directive meaning and this interpretation is restricted by the postulates of deontic logic [the logic of directives]. It follows that the formula F(O(*p*)) & F(O(¬*p*)) is logically unimpeachable if it is taken to mean only that *A* has uttered the words "Pick up the hat and leave it where it is." It must, however, be ruled out as illegitimate if interpreted in directive terms as stating that *A* has given *B* the order pick up the hat and leave it where it is.

According to this conception of the logic of directives and the logic of normative propositions, the fact that some norm-authority can utter conflicting directives does not mean that the corresponding normative propositions should be regarded

as logically compatible. In other words, if *A*'s utterance is described as a *directive* (as having "directive meaning"), the logic of F-propositions (normative statements)[23] depends on the logic of directives, that is, on what Ross regards as deontic logic proper. This view is consistent with the view that normative and descriptive utterances of normative sentences have the same "directive meaning", and this meaning determines the logical properties of both kinds of utterances. Moreover, as was observed above, Ross's characterization of a directive (a command) as a normative act which "prescribes an obligation" to act in a certain way supports this account of the semantics of directives.

In the light of the semantics of SDL outlined in Section 1, it is easy to see how a system which contains the directives (normative propositions) **O**A and **O**¬A fails "to carry out the function of directing human behavior". A system of directives guides a norm-subject's behavior by dividing the possibilities (courses of action) open to him into normatively acceptable and unacceptable possibilities. A system which contains conflicting directives fails to do this, because it makes all possibilities normatively unacceptable.

According to Carlos Alcourrón and Eugenio Bulygin (1993, pp. 283–84), SDL can be regarded as a logic of norms (the logic of "prescriptive normative operators"), but not as a logic of normative statements (the logic of "descriptive normative operators"). In SDL, the concept of permission is defined by (P), as the absence of a prohibition. Alcourrón and Bulygin argue that on the level of normative statements, it is possible to distinguish two concepts of permission which should be represented by different permission operators. A state of affairs or an act A is "positively" or "strongly" permitted according to a system N if and only if N contains a norm by virtue of which A is permitted or which makes A permitted. (Alcourrón and Bulygin 1993, pp. 286–87, Alchourrón 1969, pp. 249–50.) Alcourrón and Bulygin call the absence of prohibition the "negative" or "weak" concept of permission, and argue that the two concepts are not equivalent: the fact that a system contains no prohibition against A does not make it "positively" permitted; a norm system may also contain explicit permissions.

Ross rejects the attempt to make such a distinction. A permission is simply a negation of an obligation; thus Ross accepts schema (P) (see section 1) as a definition of the concept of permission (1968, pp. 120–21). Concerning legal norms, he observes (1968, p. 122):

[23] 'F' refers here to the "indicative" associated with a directive.

> I have never heard of any law's being passed with the purpose of declaring a new form of behavior (e.g., listening to the wireless) permitted. If a legislator sees no reason to interfere by issuing an obligating prescription (a command or a prohibition) he simply keeps silent. I know of no permissive legal rule which is not logically an exception modifying some obligation, and interpretable as the negation of an obligation.[24]

Ross's remark about the nature of permissive legal rules suggests that the distinction between negative and positive permission is based on a confusion between the meaning of permission sentences and the possible effects of the performative utterances of such sentences. When a permission sentence is used performatively, the utterance changes the addressee's obligations and expands the field of permissibility (for example) by introducing an exception to some pre-existing mandatory norm (an obligation or a prohibition). A performative utterance of a permission sentence changes the normative system, but as Ross emphasizes, the content of the utterance is the negation of an obligation or a prohibition, that is, the same as that of a "negative" permission. A normative utterance of a permission sentence is a "positive" act in the sense that it can change the options of the addressee, but this does not mean that the concept of permission used in such a normative act differs from that used in the assertoric utterances of permission sentences.[25]

According to the thesis that the logic of norms is the same as the logic of normative propositions, the validity conditions of norms are the truth-makers of normative propositions. The validity conditions of norms and directives depend on the kind of directive under consideration. In the case of simple imperatives

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[24] Ross's criticism is directed against G. H von Wright's distinction between "weak" and "strong" permission (von Wright 1963, pp. 85–92), but it applies to Alchourrón and Bulygin's distinction between negative and positive permission.

[25] In their defense of the distinction between weak and strong permission Alchourrón and Bulygin (1984, 369–70) observe that if there are several hierarchically ordered normative authorities and a high authority K issues a permissive norm **P**A, K may thereby make a lower authority, say M, incapable of prohibiting the act or state of affairs A, and thus limit the competence of the lower authority. In such a situation K's permissive norm "can be interpreted as a rejection in advance" of the corresponding (possible) prohibition to be issued by M. In this way normative utterances of permission sentences can change a complex norm system by limiting the powers of the inferior norm authorities, and not only by creating exceptions to mandatory norms. However, *as a permission*, the content of such an utterance is logically indistinguishable from a "weak" permission, that is, the negation of an obligation.

(commands), we may assume that the utterance of an imperative, for example, "Peter, mail this letter!" is enough to make it valid and the corresponding normative proposition 'Peter is required to mail this letter' true; in this case "saying makes it so" (Lewis 1979, p. 166). In the case of legal norms the question about norm validity is more complex: the mere utterance of a normative proposition does not ensure the validity of the norm (or directive) and the truth of the corresponding normative proposition if the utterer does not have the competence to issue the norm in question. The question about the validity conditions of legal norms is one of the central questions of legal philosophy, but it is not a question for the logic of norms; in the logic of norms it is presupposed that normative utterances can be valid or invalid.

4. Ross on the Logic of Norm Validity

According to Ross, the "logical element" of a directive refers to its validity (Ross 1941, pp. 58, 62), and the logic of validity differs from the logic of satisfaction: in the logic of norm-validity, the directive **!**(A $\vee$ B) cannot be inferred from **!**A. The validity conditions of directives determine (or are the same as) the truth-conditions of normative propositions; therefore, according to Ross, the deontic principle (1.2) is not logically valid. In this way we arrive at the formulation of Ross's paradox (or the problem of disjunctive obligation) presented in the first section of this paper.

In his book (1968) Ross presents an overview of various deontic principles, considered as principles of norm validity. In his discussion of W. N. Hohfeld's (1919) "fundamental legal conceptions" he calls the concepts of obligation, permission, claim, no-claim, competence, immunity, and related notions "legal-directive modalities" (1968, ch. V, p. 118; Ross 1958, pp. 158–169), but in the chapter on deontic logic (the logic of directives) (1968, ch. VI), he does not regard deontic logic as a branch of modal logic (p. 182 n. 1).[26]

Ross considers both "internal" and "external" uses of logical connectives. A propositional connective is used externally when it does not occur in the scope of a deontic operator; internal use is the use of a connective in the scope of a deontic operator. The valid principles (or schemata) which do not contain any

[26] Ross (1968, p. 182 n. 1) makes a curious remark about modal logic and deontic logic. He notes that Stenius (1963) and von Wright regard deontic logic as a branch of modal logic, and suggests that von Wright's deontic logic is "partly a modal logic, partly a calculus of directives", but he thinks that "the two fields should be kept as separated in deontic as they are in indicative logic." (By "the two fields" he apparently means modal logic and non-modal (extensional) logic.)

internal occurrences of connectives are simply logical truths of extensional logic (propositional logic and quantification theory), and there is not need to consider such principles here.[27] Ross accepts the following schemata as valid principles of normative reasoning; they are formulated here in the notation I have used above for deontic propositions (1968, p. 168):

(4.1) $\mathbf{O}A \rightarrow \neg\mathbf{O}\neg A$ (= (OD))
(4.2) $\mathbf{O}(A \,\&\, B) \rightarrow \mathbf{O}A \,\&\, \mathbf{O}B$
(4.3) $\mathbf{O}(A \vee B) \rightarrow \neg\mathbf{O}A \,\&\, \neg\mathbf{O}B$
(4.4) $(\mathbf{O}(A \rightarrow B) \,\&\, A) \rightarrow \mathbf{O}B$.

Moreover, Ross also accepts the converse of (4.2), the "agglomeration principle"

(4.5) $\mathbf{O}A \,\&\, \mathbf{O}B \rightarrow \mathbf{O}(A \,\&\, B)$;

thus he accepts the conjunction principle

(4.6) $\mathbf{O}(A \,\&\, B) \leftrightarrow \mathbf{O}A \,\&\, \mathbf{O}B$.

(Ross 1968, p. 163.) Schemata (4.2) and (4.5) (and hence (4.6)) are valid in the system D of deontic modalitites, that is, in the standard system of deontic logic, but (4.3) and (4.4) are not valid in SDL.

Ross regards the following schemata as invalid (1968, pp. 160, 166):

(1.2) $\mathbf{O}A \rightarrow \mathbf{O}(A \vee B)$
(4.7) $\mathbf{O}(A \rightarrow B) \rightarrow (\mathbf{O}A \rightarrow \mathbf{O}B)$. (= (OK))[28]

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[27] Ross uses the connectives $\neg$, $\&$, $\vee$, and $\rightarrow$ in their usual truth-functional sense, in particular, $\rightarrow$ is sign of a material conditional. See Ross (1968), pp. 151, 160–64.

[28] The reasoning underlying Ross's rejection of (4.7) (that is, (OK)) is unclear. He assumes that Jørgensen's example of seemingly valid normative (imperative) inference

Love your neighbor as yourself!
Love yourself!
(Therefore:) Love your neighbor!

has the form $\mathbf{O}(A \rightarrow B)$, $\mathbf{O}A$; therefore $\mathbf{O}B$ (Ross 1968, p. 166).The inference has this form if the first premise is understood as the command to love one's neighbor if one loves oneself. However, Ross thinks that the inference in question is not valid, because the first premise says

Both are instances of the application of the consequence principle (ROK). In SDL, (4.7) is valid, but Ross's principle (4.4) is invalid. (4.4) is Ross's formal representation of reasoning from conditional or hypothetical norms. It expresses a principle of "factual detachment", according to which one may detach a normative conclusion **O**B from a conditional norm and a factual premise A (cf. Hilpinen 2001a, p. 171). According to (4.4), such reasoning has the form

(4.8) $\mathbf{O}(A \rightarrow B)$, A; therefore $\mathbf{O}B$.

(4.8) is a valid inference form only if

(4.9) $\mathbf{O}(A \rightarrow B) \rightarrow (A \rightarrow \mathbf{O}B)$

is a valid schema. (4.9) is (by contraposition) equivalent to

(4.10) $\neg(A \rightarrow \mathbf{O}B) \rightarrow \neg\, \mathbf{O}(A \rightarrow B)$,
that is,
(4.11) $(A \,\&\, \mathbf{P}\neg B) \rightarrow \mathbf{P}(A \,\&\, \neg B)$,

which is obviously invalid. If the principle of factual detachment is formulated by means of a material (truth-functional) conditional, the relevant conditional is

(4.12) $A \rightarrow \mathbf{O}B$,

not $\mathbf{O}(A \rightarrow B)$, and the principle of factual detachment is simply an instance of Modus Ponens. The reason why Ross formulates the relevant conditional as $\mathbf{O}(A \rightarrow B)$ rather than as $A \rightarrow \mathbf{O}B$ may be his view that the latter formula is somehow ill-formed. He says that a sentence having the form (4.12) is "an impossible hybrid, because it symbolizes neither an indicative nor a directive" (1968, pp. 167–68). However, if the distinction between indicatives (i.e., normative statements) and directives is regarded as a distinction between two kinds of utterances, as suggested above in section 3, there is no reason not to regard (4.12) and other "mixed" formulas as well-formed and meaningful.

The study of conditional norms has shown that many conditional norms cannot be represented as material (truth-functional) conditionals (cf. Hilpinen

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that "one is to love one's neighbour in the same way as one *actually* loves oneself" (1941, p. 67). This interpretation seems unwarranted, but if the first premise is understood in this way, the inference does not have the suggested form $\mathbf{O}(A \rightarrow B)$, $\mathbf{O}A$; therefore $\mathbf{O}B$.

2001a, pp. 168–73), but Ross's writings on the logic of norms belong to a period when philosophers had not yet began to study other ways of representing conditional norms, for example, their formalization in intensional (modal) logic.

According to schema (1.1), (4.3) can be written in the form

(4.13) $\mathbf{O}(A \vee B) \rightarrow \mathbf{P}\neg A \,\&\, \mathbf{P}\neg B$.

According to Ross, the inference schema

(4.14) $\mathbf{O}(A \vee B)$, $\mathbf{O}\neg A$, therefore $\mathbf{O}B$

has "incompatible" (i.e., jointly inconsistent) premises (1968, p. 161), which means that he accepts the schema

(4.15) $\mathbf{O}(A \vee B) \rightarrow \mathbf{P}A \,\&\, \mathbf{P}B$

as a valid principle of deontic logic. According to (4.13) and (4.15), the disjuncts of a disjunctive directive are always normatively indifferent, and thus a disjunctive directive is always what Rescher and Robison (1964) and Lennart Åqvist (1965) have called a "choice-offering command" (directive): it offers the addressee a free choice between the alternatives presented in the command.

(4.15) shows the connection between Ross's paradox and another putative paradox of deontic logic, the so-called paradox of free choice permission. (See von Wright 1968, pp. 21–22, Kamp 1973/74.) According to principle (OP),

(4.16) $\mathbf{O}(A \vee B) \rightarrow \mathbf{P}(A \vee B)$.

According to SDL, a disjunctive permission entails the permissibility of at least one of the disjuncts, that is,

(4.17) $\mathbf{P}(A \vee B) \rightarrow (\mathbf{P}A \vee \mathbf{P}B)$,

but not the permissibility of both;

(4.18) $\mathbf{P}(A \vee B) \rightarrow (\mathbf{P}A \,\&\, \mathbf{P}B)$

is not a theorem of the standard system. However, a disjunctive permission often seems to make both disjuncts permitted. For example, the permission

(4.19) You may drink beer or wine

seems give the addressee the permission to drink beer and the permission to drink wine (but not necessarily the permission to drink both beer and wine), thus (4.19) seems to entail

(4.20) You may drink beer and you may drink wine.

(Cf. von Wright 1968, p. 21.) However, this inference is not justified by the standard deontic logic. On the other hand, according to the principle

(4.21) $\mathbf{P}A \rightarrow \mathbf{P}(A \vee B)$

of SDL, the permission statement

(4.22) You may drink beer

entails the disjunctive permission (4.19), which seems counter-intuitive in the same way as the Ross formula (1.2), and leads to an absurd result if one then proceeds to derive (4.20) from (4.19).

This puzzle about disjunctive permission has led some philosophers to distinguish between two concepts of permission, *weak* and *strong* or explicit permission. It has been suggested that schema (4.21) holds for the weak permission, which means just an absence of an obligation to do the opposite, whereas the concept of strong (or explicit) permission satisfies principle (4.18). (von Wright 1971, 160, 164–65; cf. Åqvist 1987, pp. 46, 52–53.) However, as was observed above, Ross rejects such a distinction: to say that A is permitted means that $\neg A$ is not required (or obligatory), and nothing more. To postulate a special concept of "strong" permission to account for the puzzle of free choice permission seems an *ad hoc* solution to the problem. Schema (4.21) is obviously a valid conditional if the semantics of P-sentences is understood in the way outlined at the end of Section 1. If Peter drinks beer in some deontically acceptable (or legal) situation, then he drinks beer or wine in some such situation.

Ross accepts schema (4.2), but rejects (1.2). Both are instances of the consequence principle (ROM); thus he seems to be committed to the view that the consequence principle is not a valid principle of normative reasoning, but has some valid instances. Moreover, according to the rule (ROE), the principle

of intensionality (intersubstitutability of logically equivalent propositions), the antecedent of (2.1), **O**A, can be written as

(4.23) $\mathbf{O}((A \vee B) \,\&\, (A \vee \neg B))$,

and according to (4.2), (4.23) entails

(4.24) $\mathbf{O}(A \vee B)$,

which means that the problematic schema (1.2) rejected by Ross appears to be valid after all. Moreover, according to the principle of contraposition, Ross's "free choice principle" (4.3),

$\mathbf{O}(A \vee B) \rightarrow \neg\mathbf{O}A$,

can be reformulated as

(4.25) $\mathbf{O}A \rightarrow \neg\mathbf{O}(A \vee B)$,

that is,

(4.26) $\mathbf{O}A \rightarrow \mathbf{P}\neg(A \vee B)$.

This is a strongly counter-intuitive schema. For example, according to (4.26), if Peter is required to tell the truth, then he may neither tell the truth nor (say) pay his debts. Moreover, schemata (4.1) and (4.6), which Ross accepts as valid, and rules (RO) (the rule of "deontic necessitation") and (ROE) (the substitutability of provably equivalent sentences in deontic contexts) form a sufficient axiom system for the standard system of deontic logic, and therefore validate the principles rejected by Ross, viz., (1.2) and (4.7). (Cf. Føllesdal and Hilpinen 1971, pp. 12–13.)

It is possible to develop a logic of directives (or commands) in which the Ross formula (2.1) is not valid, but in such a logic the conjunction principle (4.6) is not valid either. Krister Segerberg (1990) has constructed for directives a "logic of validity" (as opposed to the "logic of satisfaction") in which (2.1) and the conjunction principles (4.2) and (4.6) are invalid. He analyzes directives by means of a directive operator **!** and an action operator **do** which turns propositions into action descriptions. A formula '**do**A' may be read: '(to) bring it about that A', and '**!do**A' expresses a directive or a command to bring it about that A or see to it that A, and (according to Segerberg) can be read: "Do anything to bring it about that A". If '**O**A' is used as an abbreviation of '**!do**A',

neither (1) nor (13) is valid in Segerberg's semantics for directives. He argues that if the O-sentences are understood in this way, this is a plausible result (Segerberg 1990, p. 217):

> If I have been commanded to do anything to bring it about that A & B, it does not follow that I have been commanded to do anything to bring it about that A. Notice that this is the same kind of argument we use to resist the formula Ross found objectionable: if I have been commanded to do anything to bring it about that A, it does not follow that I have been commanded to do anything to bring it about that A ∨ B.

Segerberg's defense of Ross's position is not entirely convincing. 'To bring it about that A' can be taken to mean 'Do something which is sufficient for the truth of A', and this reading of '**O**A' seems to support (ROM), (2.1), and (4.2). On the other hand, if '**do**A' is taken to refer to an action which is both sufficient and necessary for A, (ROM), (2.1) and (4.2) are invalid. (See Kanger 1972, pp.108–110, 121–22; Hilpinen 2001b, p. 141). Regardless of whether the Ross formula (2.1) is accepted as valid, it seems clear that it and (4.2) stand or fall together (Segerberg 1990, loc.cit.). However, as we have seen, Ross accepts both (4.2) and (4.5). He supports the equivalence (4.6) (the conjunction of (4.2) and (4.5)) as follows:

> It seems to come to the same things whether it is said in the watchman's instructions: "At closing time the following duties are incumbent on the watchman: (1) closing the gate; (2) turning the dog loose; and (3) putting out the light."
> Or whether the instructions say: "At the closing time it is the watchman's duty to close the gate, to turn the dog loose, and to put out the light." (Ross 1968, p. 163.)

(2.1) and (4.2) can be pulled logically apart only be giving up principle (ROE) (the intersubstitutability of logically equivalent expressions in deontic contexts), for example, by assuming that **O**(A) does not have the same content as **O**((A ∨ B) & (A ∨ ¬B)). However, this would mean that in deontic contexts, the connectives 'or' (∨) and 'and' (&) do not have their customary truth-functional meaning or only their truth-functional meaning.[29] It seems

[29] Jennings (1994) is a detailed study of the behavior and functions of the word 'or' in various contexts, including deontic and other modal contexts; see pp. 117–157.

difficult to systematize Ross's conception of valid directive reasoning in the form of a coherent logical system. This suggests that it may be advisable to approach Ross's puzzle in a different way.

5. The Content of Norms and the Performative Effects of Normative Utterances

There are cases in which the apparent inconsistency of an utterance is not due to an inconsistency in its content, that is, in the proposition uttered, but depends on other features of the utterance. The so-called Moore's paradox of saying and disbelieving is a well known example of such an utterance. (See Moore 1912/1947, p. 78; 1942/1968, p. 541; 1993, pp. 209–210.) For example, the assertion

(5.1) It is raining, but I do not believe that it is raining.

may seem inconsistent. The proposition uttered in (5.1) has the form

(5.2) $A \,\&\, \neg \mathbf{B}_{\mathrm{I}} A$,

where $\mathbf{B}_{\mathrm{I}}$ represents a first-person belief. It is clear that (5.2) cannot be regarded as an inconsistent propositional form, but by asserting (5.1) the speaker can be said to contradict himself. An utterance or assertion

(5.3) $\mathbf{As}(A \,\&\, \neg \mathbf{B}_{\mathrm{I}} A)$

is "contradictory" in the following sense: An assertion of a conjunctive proposition is an assertion of both conjuncts; thus (5.3) is equivalent to the conjunction of

(5.4) $\mathbf{As}A$
and
(5.5) $\mathbf{As}\neg \mathbf{B}_{\mathrm{I}} A$.

A sincere assertion is an expression of belief: by asserting a proposition A the utterer *states* that A and *expresses* the opinion that A (*conveys* the information or *implies* that he believes that A); thus (5.4) conveys the information that the speaker believes that A, and in the latter assertion (5.5) the speaker *asserts* that he does not believe that A. (See Moore 1912/1947,

pp. 78–79; 1942/1968, pp. 540–43.) The two assertions cannot be "correct" at the same time: either the first is not sincere, or the second is false. This can be regarded as an instance of "contradicting oneself". In this sense anyone who utters a Moore sentence contradicts himself. The indefensibility of an assertive utterance of a Moore sentence can be explained without assuming that the speaker utters a contradictory proposition.

An analogous strategy can be applied to the deontic inferences which Ross regards as problematic. Let us consider Ross's view that schema (4.3) is logically valid, that is, that **O**(A ∨ B) and **O**A are jointly inconsistent. According to Ross, this is

> due to the fact that internal disjunction [i.e., **O**(A ∨ B)] expresses a freedom of choice which is incompatible with a choiceless duty toward any of its constituent parts or toward both of them. (Ross 1968, p. 160.)

In the light of Moore's example, we might say that an *utterance* (especially a performative utterance) of **O**(A ∨ B) can in suitable circumstances express a freedom of choice between A and B and the permissibility of both disjuncts. This may be the case even if **O**(A ∨ B) does not *state* (that is, entail) that the addressee is free to choose either one of the two alternatives. The freedom of choice may be expressed or conveyed by an utterance of a normative statement without being part of the content of the statement itself. The situation can be regarded as analogous to that in Moore's example: an assertive utterance of A expresses the speaker's belief that A, but the proposition that A itself does not express such a belief.

This way of solving Ross's puzzle about disjunctive directives resembles a proposal made by Erik Stenius (1982). Stenius makes a distinction between a normative system (system of directives) and its codification or *codex*: a normative system S can be *given* or presented in different ways. (Stenius prefers to use the word 'codex' instead of the more common '(normative) code' (1982, p. 73, n. 12); I shall use here the latter expression.) The code C_S of a system S consists of the *formulated* directives of S; the system S can be defined as the set of logical consequences of C_S. The formulation of a legal code is subject to certain tacitly accepted rules which Stenius calls rules for "well-formed codices" (1982, pp. 73–74). For example, a good normative code should not contain a directive which is a logical consequence of another directive in the code:

(WfC.1) A normative code C is well-formed only if it does not contain any directive which is a logical consequence of another directive in C.

(WfC.1) is a special case of the following more general rule:

(WfC.2) A normative code C is well-formed only if it does not contain any directive which is a logical consequence of a number of other directives in C.

These rules reflect the view that an optimal codification should consist of logically independent directives. The rules for well-formed normative codes might also include the rule that the formulated directives should correspond to the reasons or grounds on the basis of which they have been adopted. (See Hilpinen 1981, p. 161.) According to (WfC.1) and (WfC.2), a code which contains a directive **O**A should not contain the disjunctive directive **O**(A $\vee$ B):

> If we presuppose that "saying" **O**(A $\vee$ B) and nothing else means that the regulation forms a complete codex of a system S then we can infer that neither **O**A, **O**B, **O**$\neg$A nor **O**$\neg$B can belong to the *system*, since **O**(A $\vee$ B) does not entail any of these obligations. Thus A, $\neg$A, B, and $\neg$B are permitted in S. (Stenius 1982, p. 75.)

Stenius observes that the problematic inferences "are inferences from assumptions about what 'saying' something implies in a certain context, not from one obligation [directive] to other norms," and suggests that the fallacy underlying Ross's schemata (4.13) and (4.15) is a confusion between "an inference 'ex silentio' and logical inference." (1982, p. 75.) This fallacy is the same as the confusion between what a statement entails and what an utterance of a statement expresses or conveys in a certain context.[30]

As was observed earlier, the function of the performative use of O- and F-sentences is to contract the agent's field of permissibility, i.e., the range of options open to the agent. Consider Ross's example about posting a letter. The directive **O**A ('Mail the letter!') contracts the agent's possibilities by excluding all possibilities (possible courses of action) in which the agent fails to mail the letter. These possibilities include the possibilities in which the

[30] Paul Grice has formulated a similar distinction in terms of the concept of "conversational implicature" as a distinction between what a speaker says and what he "conversationally implicates" in a certain context; cf. Hare (1967), p. 311; Grice (1975/1989), pp. 24–26.

agent not only fails to mail the letter, but also burns it. In this situation, the directive 'Mail or burn the letter!' would have no effect on the field of permissibility, because the possibilities excluded by it (those in which the agent neither mails nor burns the letter) have already been excluded by the previous command to post the letter. However, an utterance of the disjunctive command would be normatively significant if it were regarded as an act of replacing the earlier directive to post the letter in the code of directives given to the agent by a new directive which lets the agent choose between mailing and burning the letter. According to this interpretation, an utterance of the second directive is a normative act which partly derogates or cancels the earlier obligation to post the letter and not burn it. This interpretation is an attempt to construe the utterance 'Post or burn the letter!' as a significant normative act, but it is important to note that it is based on the view that the Ross formula (1.2) holds for normative propositions and the deontic proposition that the letter is to be mailed or burned does not add any normative content to the proposition that the letter has to be mailed.

It is clear that the effects of a normative utterance of $\mathbf{O}(A \vee B)$ are usually different from the effects of an utterance of $\mathbf{O}A$. The view that this counts as evidence against the validity of (2.1) seems to be tacitly based on the following assumption:

(5.6) If a directive D_1 entails D_2, then the normative (performative) effects of D_1 entail (are included among) the effects of D_2.

This assumption is obviously false: logical deduction preserves truth, but not information, and the effects of a directive depend on the information conveyed by the directive and its utterance (the fact that the directive has been uttered). Deductive inference cannot be expected to preserve the normative effects of a directive any more than logical deduction preserves the effects of the acceptance of a declarative statement on a person's belief system or corpus of knowledge.

Similar considerations apply to disjunctive permission sentences. The function of a performative utterance of a permission sentence is to expand the field of permissibility by derogating some obligation. According to SDL, $\mathbf{P}(A \vee B)$ means that some possibility (possible course of action) in which $A \vee B$ holds is normatively acceptable. If an agent is originally prohibited from drinking beer (A) and prohibited from drinking wine (B), that is, in a situation in which

(5.7) $\neg\mathbf{P}A \,\&\, \neg\mathbf{P}B$

holds, a normative utterance of $\mathbf{P}(A \vee B)$ makes it permitted to drink beer or wine. We can distinguish here four possible outcomes of such an utterance:

(5.8.1) $\mathbf{P}A \,\&\, \neg\mathbf{P}B$ (The agent may drink beer but not wine.)
(5.8.2) $\neg\mathbf{P}A \,\&\, \mathbf{P}B$ (The agent may drink wine but not beer.)
(5.8.3) $\mathbf{P}A \,\&\, \mathbf{P}B \,\&\, \neg\mathbf{P}(A \,\&\, B)$ (The agent may drink beer and may drink wine but not both.)
(5.8.4) $\mathbf{P}(A \,\&\, B)$ (The agent may drink both beer and wine.)

(5.8.1)–(5.8.4) represent the possible ways in which $\mathbf{P}(A \vee B)$ can be true. Which of these situation types results from the utterer's normative act? It is clear that this is not determined by the content or the meaning of the utterance $\mathbf{P}(A \vee B)$ alone, but depends on other considerations. If the outcome is (5.8.1) or (5.8.2), the authority has removed one of the two prohibitions, $\mathbf{F}A$ or $\mathbf{F}B$ (i.e., $\neg\mathbf{P}A$ or $\neg\mathbf{P}B$), but to do this in an effective way, he should obviously have given the corresponding permission, $\mathbf{P}A$ or $\mathbf{P}B$, and not a disjunctive permission.[31] On the other hand, in saying that the addressee may drink beer or wine, the authority does not necessarily give the permission to drink both; to permit that possibility, he should have said that the addressee may drink both beer and wine. If a normative utterance of $\mathbf{P}(A \vee B)$ leads to a determinate outcome, that outcome should presumably be (5.8.3), that is, a situation in which the addressee may drink beer and may drink wine, but not both. The utterance is compatible with the prohibition against $A \,\&\, B$, and thus leaves that prohibition intact.[32] Usually a performative utterance of a disjunctive permission sentence expresses the permissibility of both disjuncts, but this does not presuppose the validity of schema (4.18). This can be seen from the fact that the implication that both A and B are permitted can be canceled without contradiction. The normative authority can do this by saying, for example: "You may drink beer or wine, but before you drink wine, consult your physician." (See Kamp 1979, p. 271.) In this case the utterance of the disjunctive permission sentence leaves the

[31] Anthony Kenny has made a similar observation about Ross's example. In a situation in which the directive $\mathbf{O}A$ would be "satisfactory" for a certain purpose, the directive $\mathbf{O}(A \vee B)$ might not be satisfactory for that purpose; therefore the former does not entail the latter in the "logic of satisfactoriness". (Kenny 1966, pp. 72–74; 1975, pp. 80–83.)

[32] The conjunction of $\mathbf{F}A$ and $\mathbf{F}B$ entails $\mathbf{F}(A \,\&\, B)$.

field of permissibility partly indeterminate: the authority makes the disjunction A ∨ B permitted for the addressee, and leaves further determination of the addressee's field of permissibility to another normative authority. A performative utterance of a permission sentence may fail to change a normative system in a determinate way, that is, lead to a well-defined revised system. (Cf. Bulygin and Alchourrón (1977), pp. 29–30; Lewis (1979), pp. 167–175; Hilpinen (1981), pp. 158–160). A permissive utterance has a determinate outcome only if it is chosen in such a way that it transforms a given normative code into a new, well-defined code. According to Hans Kelsen, derogating norms should contain explicit reference to the norm to be repealed (1973, p. 263). The problem of indeterminacy can be avoided in this way if the norm to be derogated is part of the code of the system so that the act of derogation leads to a well-defined normative code.

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